

Closing Fri: 3.4(part 1),3.4(part 2)

Closing Mon: 10.2

Closing Wed: 3.5(part 1)

Closing Next Fri: 3.5(part 2)

Exam 1 will be returned Tuesday.

Grades will be posted by the end of next week.

### 3.4 Chain Rule

The **composition** of two function is defined by

$$(f \circ g)(x) = f(g(x))$$

*Example:*

If  $f(x) = \sin(x)$ ,  $g(x) = x^3$ , then

$$(f \circ g)(x) = \sin(x^3).$$

The derivative of a composition:

**Chain Rule:**

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

Also written as:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

*Example:*

$$\frac{d}{dx} \sin(x^3) = \cos(x^3) 3x^2$$

Brief “proof sketch” for the chain rule:

From the definition of derivative

$$\begin{aligned} & \frac{d}{dx} f(g(x)) \\ &= \lim_{h \rightarrow 0} \frac{f(g(x+h)) - f(g(x))}{h} \\ &= \lim_{h \rightarrow 0} \left( \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \right) \left( \frac{g(x+h) - g(x)}{h} \right) \\ &= f'(g(x))g'(x) \end{aligned}$$

*Examples:* Find the derivative

1.  $y = (2x^2 + 1)^2$

2.  $y = e^{\sin(2x)}$

3.  $y = \tan(3x + \cos(4x))$

4.  $y = \sin^4(x)$

5.  $y = \sin(x^4)$

Identify the first rule you would use to differentiate these functions:  
(sum, product, quotient or chain)

$$1. y = \sqrt{\sin(x) + x^2 + 1}$$

$$2. y = \frac{x^4}{\sin(5x+1)}$$

$$3. y = \sqrt[3]{4x + 1} \cos(\sin(2x))$$

$$4. y = e^{\tan(x)} - 5(x^8 + 1)^{50}$$

$$5. y = \left( \frac{x^2 - 1}{x^4 + 1} \right)^{10}$$